

INSTANTONS AND MONOPOLES IN LATTICE QCD

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Abstract

We analyze the interplay of topological objects in four-dimensional QCD on the lattice. The distributions of color magnetic monopoles in the maximum abelian gauge are computed around instantons in both pure and full QCD. We find an enhanced probability for monopoles inside the core of an instanton on gauge field average. This feature is independent of the topological charge definition used. For specific gauge field configurations we visualize the situation graphically. Moreover we investigate how monopole loops and instantons are correlated with the chiral condensate. Strong evidence is found that clusters of the quark condensate and topological objects coexist locally on individual configurations.

1 Introduction and Theory

There are two different kinds of topological objects which seem to be important candidates for the confinement mechanism: color magnetic monopoles and instantons. In lattice calculations we demonstrated that color magnetic monopoles and instantons are correlated on realistic gauge field configurations [1]. Similar phenomena were discussed by other groups on semiclassical configurations [2]. This might indicate that both confinement mechanisms have the same topological origin and that both approaches can be united. It is believed that instantons and also monopoles can explain chiral symmetry breaking [3, 4]. In this contribution we present first results on the local correlation of the chiral condensate $\bar{\psi}\psi(x)$, the topological charge density $q(x)$, and the monopole density $\rho(x)$ on single gauge fields.

To investigate monopole currents we project $SU(N)$ onto its abelian degrees of freedom, such that an abelian $U(1)^{N-1}$ theory remains [5]. We employ the so-called maximum abelian gauge being most favorable for our purposes. For the definition of the monopole currents $m_i(x, \mu)$, $i = 1, \dots, N$, we use the standard method [6]. From the monopole currents we define the local monopole density as $\rho(x) = \frac{1}{4NV_4} \sum_{\mu,i} |m_i(x, \mu)|$.

There exist several definitions of the topological charge on the lattice. We use a field theoretic and a geometric charge definition. The field theoretic prescriptions are a straightforward discretization of the continuum expression. To get rid of the renormalization constants we apply the “Cabbibo-Marinari cooling method” which smooths the quantum fluctuations of a gauge field. The geometric charge definitions interpolate the discrete set of link variables to the continuum and then calculate the topological charge directly. In our studies of the topological charge density $q(x)$ we employ the hypercube and plaquette prescription for the field theoretic definition [7] and the locally gauge invariant Lüscher charge definition [8].

To measure correlations between topological quantities we calculate functions of the type

$$\begin{aligned} \langle q(0)q(d) \rangle, \langle \rho(0)\rho(d) \rangle, \langle \rho(0)q^2(d) \rangle, \\ \langle q^2(0)\bar{\psi}\psi(d) \rangle, \langle \rho(0)\bar{\psi}\psi(d) \rangle. \end{aligned} \quad (1)$$

They are normalized after subtracting the corresponding cluster values.

2 Results

In Fig. 1 the correlation functions between the monopole density and the absolute value of the topological charge density for the hypercube and the Lüscher definition are depicted in the confinement (l.h.s.) and the deconfinement phase (r.h.s.) of pure $SU(2)$ theory on a $12^3 \times 4$ lattice with periodic boundary conditions for several cooling steps. Both definitions yield qualitatively the same result. For each charge definition the correlation functions are almost independent of cooling and extend over approximately two lattice units. This indicates that there exists a nontrivial local correlation between these topological objects and that the probability for finding monopoles around instantons is clearly enhanced. The normalized $\rho|q|$ -correlations seem to be hardly influenced by the phase transition. To gain a more quantitative insight, we analyze the correlation functions discussed above by fitting them to an exponential function. The resultant screening masses in lattice units are presented in Table 1 in both phases for 5 and 20 cooling steps. The screening masses computed from the correlations between monopoles and instantons turn out to be relatively insensitive to cooling and to the phase transition. The error bars of the masses are large reflecting the large errors in the raw data not shown for clarity of plots.

The correlation functions between topological quantities in pure $SU(3)$ theory on an $8^3 \times 4$ lattice (hypercube definition for q) are shown in Fig. 2 for several cooling steps. The range of the instanton autocorrelation qq (a) being

originally δ -peaked grows rapidly with cooling reflecting the occurrence of extended instantons. In contrast the $\rho\rho$ -correlation (b) decreases since monopole loops become dilute with cooling. The ρq^2 -correlation (c) seems rather insensitive to cooling and clearly extends over more than two lattice spacings, indicating some nontrivial local correlation between monopoles and topological charges.

Figure 3 shows correlation functions of full $SU(3)$ QCD with 3 flavors of Kogut-Susskind quarks of equal mass $ma = 0.1$ in the confinement region. The ρq^2 -correlation (a) looks similar to the corresponding function in pure QCD (Fig. 2c). The same holds for the instanton and monopole autocorrelation functions (not shown). In the case of the $\bar{\psi}\psi q^2$ -correlation (b) exponential fits show that an increasing number of cooling steps results in a narrower correlation function. The $\bar{\psi}\psi\rho$ -correlation (c) on the other hand is not sensitive to cooling and has approximately the same exponential decay as the $\bar{\psi}\psi q^2$ -correlation after some cooling steps.

We now visualize distributions of topological quantities from individual gauge fields. Fig. 4 depicts for a fixed time slice the location of a single instanton (dots) which was put on a trivial gauge field configuration artificially. Here a purely spatial monopole loop surrounds the instanton (closed line).

To obtain some insight into the topological correlations, Fig. 5 presents a cooling history of an $SU(2)$ gluon field at a fixed time slice on a $12^3 \times 4$ lattice. The topological charge density using the plaquette and the hypercube definition is displayed for cooling steps 0, 15 and 25. A dot is plotted if $|q(x)| > 0.01$. The lines represent the monopole loops. Without cooling the topological charge distribution cannot be resolved from the quantum fluctuations. Also the monopole loops do not exhibit a structure. After 15-20 cooling steps one can identify clusters of topological charge with instantons. At cooling steps 35-40 the instanton and antiinstanton begin to approach each other until they annihilate several cooling steps after (not shown). Monopole loops also thin out with cooling, but they survive in the presence of instantons. In general, there is an enhanced probability that monopole loops exist in the vicinity of instantons.

We turn to the visualization of distributions of the chiral condensate and topological quantities. In Fig. 6 a time slice of a typical configuration from $SU(3)$ theory with dynamical quarks on an $8^3 \times 4$ lattice is shown. We display the instanton density by dark dots if the absolute value $|q(x)| > 0.003$. The chiral condensate is indicated by light points if a threshold for $\bar{\psi}\psi(x) > 0.066$ is exceeded. Monopoles are represented by lines. For clarity of the plots we give only one color type of monopole currents, $m_1(x, \mu)$. By analyzing dozens

of gauge field configurations we found the following results. The topological charge is covered by quantum fluctuations and becomes visible by cooling of the gauge fields. For 0 cooling steps no structure can be seen in $q(x)$, $\bar{\psi}\psi(x)$ or the monopole currents, which does not mean the absence of correlations between them. After 5 cooling steps clusters of topological charge and chiral condensate are resolved. This particular configuration possesses a positive and a negative topological cluster corresponding to an instanton and an antiinstanton, respectively. For more than 10 cooling steps both topological charge and chiral condensate begin to die out and finally vanish.

Figure 6 suggests that the chiral condensate attains its maximum values at the same positions as the maximum values of the topological charge. This behavior is more clearly shown in Fig. 7 where the $\bar{\psi}\psi(x)$ -values are plotted against $q(x)$ for all points x in the same configuration at 10 cooling steps. Our simulations in the deconfinement phase ($\beta = 5.4$) showed very little topological activity. In less than one percent of the gauge field configurations measured, instantons could be identified safely. Also the chiral condensate is considerably lower and does not show the tendency to cluster anymore. $\bar{\psi}\psi(x)$ becomes equally distributed over the lattice points. The maximum values of $\bar{\psi}\psi(x)$ in configurations in the deconfinement after 10 cooling steps are one order of magnitude smaller.

3 Conclusion

In summary, our calculations of correlation functions between topological objects and the chiral condensate yield an extension of about two lattice spacings. The correlations suggest that the chiral condensate takes a nonvanishing value predominantly in the regions of instantons and monopole loops. With the visualization of instantons and monopole loops in specific gauge field configurations we showed directly that at the sites of instantons also monopole loops are present. This confirms our conjecture that monopoles and instantons might be two faces of a more subtle fundamental topological object, which even might carry an electric charge.

Further visualization exhibited that the chiral condensate concentrates around areas with enhanced topological activity (instantons, monopoles). To our knowledge this observation is the first direct indication that chiral symmetry breaking occurs locally in the vicinity of nontrivial topological structure. We found for full $SU(3)$ QCD with dynamical quarks that the clusters of nontrivial chiral condensate have a size of about 0.4 fm, which corresponds nicely to the instanton sizes observed in the same configurations. In the deconfine-

ment phase $\bar{\psi}\psi(x)$ is significantly lower and loses its cluster property being present in the confinement regime.

Acknowledgments

We thank M. Müller and W. Sakuler for very helpful discussions. This work was partially supported by FWF under Contract No. P11456-PHY.

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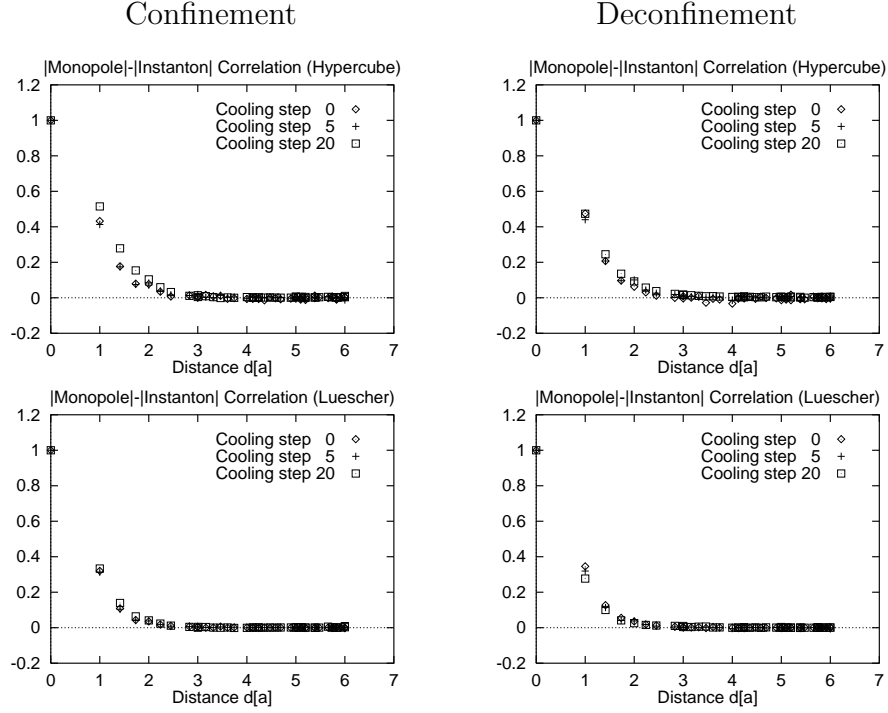


Fig. 1. Correlation functions between the monopole density and the absolute value of the topological charge density for the hypercube definition and the Lüscher definition in the confinement ($\beta = 2.25$) and the deconfinement phase ($\beta = 2.40$) of pure $SU(2)$ theory. For both definitions the monopole-instanton correlation functions are almost invariant under cooling and extend over approximately two lattice spacings. The correlation functions hardly change across the phase transition.

Table 1. Screening masses from fits to exponential decays of the correlation functions between topological objects in pure $SU(2)$ theory. The numbers in brackets are not reliable due to bad signal-to-noise ratio.

Confinement ($\beta = 2.25$)		
Correlation	Cool step 5	Cool step 20
$q - q$ (Lü)	(1.16 ± 0.59)	1.52 ± 0.40
$q - q$ (hyp)	(1.51 ± 1.12)	1.10 ± 0.48
$ q - \rho$ (Lü)	2.03 ± 0.22	2.46 ± 0.19
$ q - \rho$ (hyp)	1.86 ± 0.15	1.94 ± 0.26
Deconfinement ($\beta = 2.4$)		
Correlation	Cool step 5	Cool step 20
$q - q$ (Lü)	(1.94 ± 0.71)	1.00 ± 0.29
$q - q$ (hyp)	(2.00 ± 0.80)	1.09 ± 0.31
$ q - \rho$ (Lü)	1.69 ± 0.51	1.22 ± 0.17
$ q - \rho$ (hyp)	1.86 ± 0.32	1.56 ± 0.20

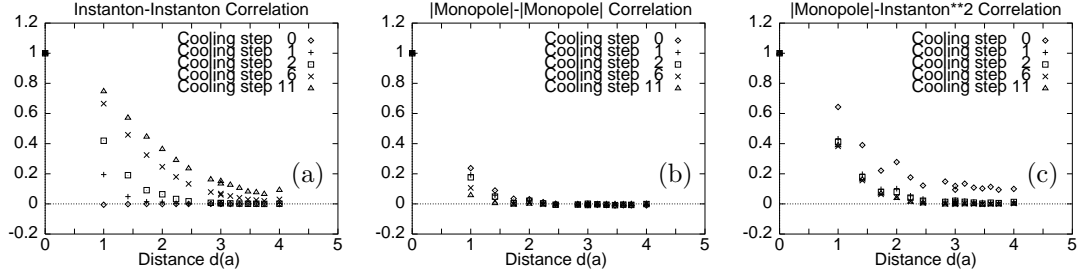


Fig. 2. Correlation functions between topological charge densities and monopole densities in the confinement phase ($\beta = 5.6$) of pure $SU(3)$ theory. The instanton autocorrelations (a) grow with cooling reflecting the existence of extended instantons whereas the monopole autocorrelations (b) decrease since monopoles become diluted. The correlations between monopoles and instantons (c) show no drastic influence from cooling and extend over approximately two lattice spacings.

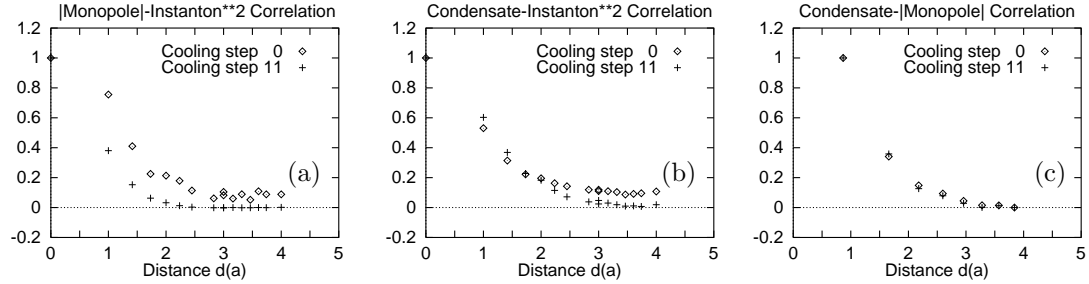


Fig. 3. Correlation functions in the presence of dynamical quarks in the confinement ($\beta = 5.2$). The monopole-instanton correlation (a) is similar as in pure $SU(3)$. The correlation of the quark condensate and the topological charge (b) is cooling dependent, whereas the correlation between the condensate and the monopole density (c) is not. All correlations extend over two lattice spacings and indicate local correlations of the chiral condensate and topological objects.

Single instanton solution

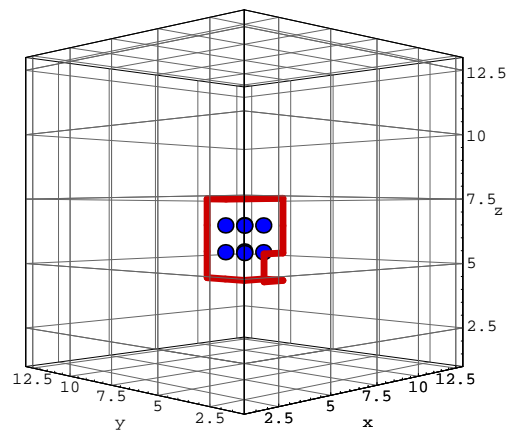


Fig. 4. Location of a single instanton (dots) at constant time imposed on a trivial gauge field configuration. A closed monopole loop (line) runs around the instanton.

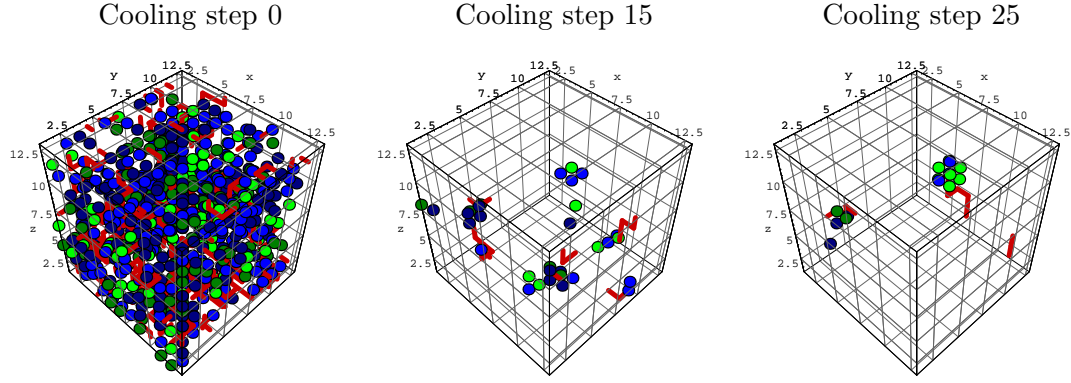


Fig. 5. Cooling history for a time slice of a single gauge field configuration of pure $SU(2)$ theory. The dots represent the topological charge distribution. Monopole loops are represented by lines. It can be seen that with cooling instantons evolve from noise accompanied by monopole loops in almost all cases. Note that black-and-white pictures do not present the situation so clearly as color plots.

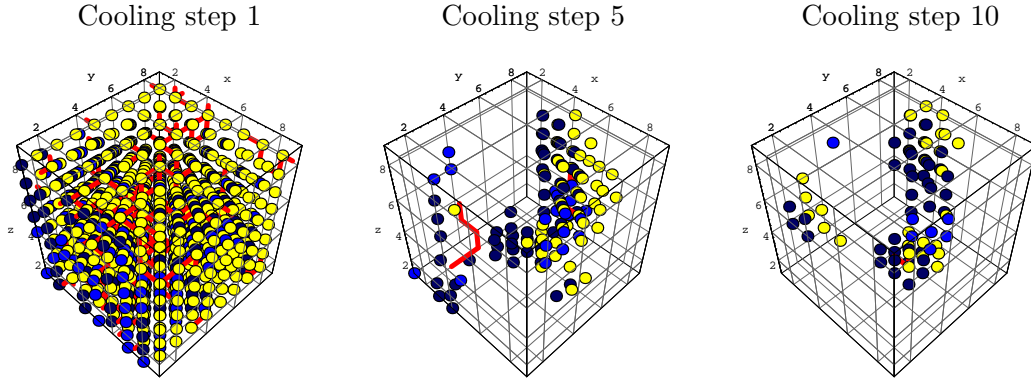


Fig. 6. Cooling history for a time slice of a single gauge field configuration of $SU(3)$ theory with dynamical quarks. The dark dots represent the topological charge distribution and the light dots the chiral condensate. Monopole loops are represented by lines. It turns out that chiral symmetry breaking occurs at the positions of the instantons.

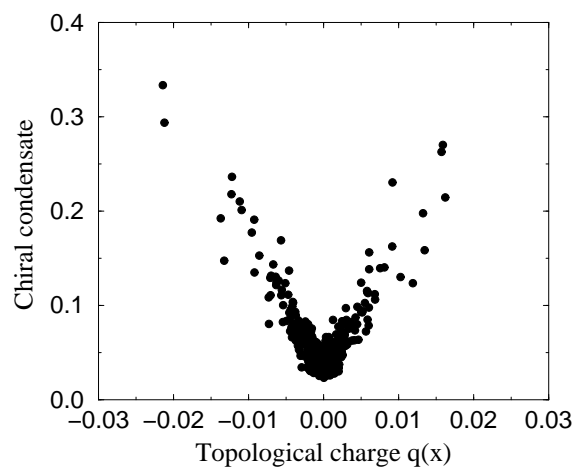


Fig. 7. Scatter plot of $\bar{\psi}\psi(x)$ against $q(x)$ in the volume of a single gauge field configuration after 10 cooling steps.